Midterm #2 – Practice

MATH 316 — Linear Algebra II
Midterm: Monday, April 13, 2026

Please print your name:

Bonus challenge. Let me know about any typos you spot in the posted solutions (or lecture sketches or any other posted material). Any typo, that is not yet fixed by the time you send it to me, is worth a bonus point.

Problem 1. Before working on the problems below, make sure that you have completed all homework.

Problem 2. Solve the initial value problem $\boldsymbol{y}' = \begin{bmatrix} 4 & -8 \\ -1 & 6 \end{bmatrix} \boldsymbol{y}, \quad \boldsymbol{y}(0) = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$.

Problem 3.

(a) Consider the following system of initial value problems:

$$\begin{aligned} y_1'' &= 3y_1' + 2y_2' - 5y_1 \\ y_2'' &= y_1' - y_2' + 3y_2 \end{aligned} \qquad y_1(0) = 1, \ y_1'(0) = -2, \ y_2(0) = 3, \ y_2'(0) = 0$$

Write it as a first-order initial value problem in the form $\boldsymbol{y}' = M\boldsymbol{y}$, $\boldsymbol{y}(0) = \boldsymbol{y}_0$.

(b) Convert the third-order initial value problem

$$y''' = 6y'' - 3y' - 10y, \quad y(0) = 1, \quad y'(0) = 2, \quad y''(0) = 3$$

to a system of first-order initial value problem.

(c) Solve the original differential equation in the previous part by solving the system.

Problem 4.

(a) What are the possible Jordan normal forms of a 6×6 matrix with eigenvalues $7, 7, 3, 3, 3, 3$?

(b) How many different Jordan normal forms are there for a 10×10 matrix with eigenvalues $8, 6, 6, 2, 2, 2, 1, 1, 1, 1$?

Problem 5. Consider the sequence a_n defined by $a_{n+2} = a_{n+1} + 6a_n$ and $a_0 = 3$, $a_1 = -1$.

(a) Determine the next two terms.

(b) Find an explicit (Binet-like) formula for a_n.

(c) Determine $\lim_{n \to \infty} \frac{a_{n+1}}{a_n}$.

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Problem 6.

(a) Find the best approximation (in the L^2 sense) of $f(x) = x$ on the interval $[0, 4]$ using a function of the form $y = a\sqrt{x}$.

(b) Find the best approximation (in the L^2 sense) of $f(x) = x$ on the interval $[0, 4]$ using a function of the form $y = a + b\sqrt{x}$.

Problem 7.

(a) The eigenvalues of a 5×5 matrix for orthogonally projecting onto a 3-dimensional subspace are ☐.

What are the eigenspaces of that matrix?

(b) Suppose A is the 3×3 matrix of a reflection through a plane (containing the origin).

Then $\det(A) =$ ☐, and the eigenvalues of A are ☐. What are the eigenspaces of A?

(c) If A has λ-eigenvalue $\boldsymbol{v}$, then A^3 has ☐.

(d) If $A = \begin{bmatrix} i & 1+2i \\ 3 & 4 \\ 5i & 6-i \end{bmatrix}$, then its conjugate transpose is $A^* =$ ☐.

(e) The norm of the vector $\boldsymbol{v} = \begin{bmatrix} 1-i \\ 2i \end{bmatrix}$ is $\|\boldsymbol{v}\| =$ ☐.

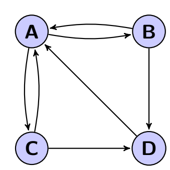
(f) If A is a reflection matrix, then $A^{-1} =$ ☐.

(g) Write down the 2×2 rotation matrix by angle θ.

(h) If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$, then $A^n =$ ☐ and $e^{At} =$ ☐.

(i) If $N^4 = \mathbf{0}$, then $e^{Nt} =$ ☐.

Problem 8. Suppose the internet consists of only the four webpages A, B, C, D which link to each other as indicated in the diagram.



Rank these webpages by computing their PageRank vector.

Problem 9. Determine an orthogonal matrix P and a diagonal matrix D such that $A = PDP^T$.

(a) If A is the 3×3 matrix for reflecting through the plane spanned by the vectors $\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

(b) If A is the 3×3 matrix for reflecting through the plane spanned by the vectors $\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

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