

Midterm #1

MATH 126-102 — Calculus II

Thursday, Sept 24, 2026

Please print your name:

No notes, graphing calculators or other tools are permitted. There are 30 points in total. You need to show work to receive full credit.

Good luck!

Problem 1. (3 points) Evaluate the following indefinite integrals.

(a) $\int \frac{dx}{3x} =$

(b) $\int \sin(4x) dx =$

(c) $\int \sqrt{x} dx =$

Solution. $\int \frac{dx}{3x} = \frac{1}{3} \ln|x| + C$, $\int \sin(4x) dx = -\frac{1}{4} \cos(4x) + C$, $\int \sqrt{x} dx = \frac{2}{3} x^{3/2} + C$

Problem 2. (5 points) Consider the region in the first quadrant bounded by the curves $y = \sqrt{x}$ and $y = x$.

Use the shell method to find the volume of the solid generated by revolving this region about the y -axis.

Solution. First, make a sketch! Note that the region extends from $x = 0$ to $x = 1$ (if this is not clear from the sketch, you can compute the intersection of $y = \sqrt{x}$ and $y = x$: $\sqrt{x} = x$ has solutions $x = 0, 1$).

Note that, if we take vertical cross-sections, then we get cylindrical shells after revolving.

The shell at position x has radius x and height $\sqrt{x} - x$. Given a thickness of dx , it therefore has (approximate) volume $2\pi x(\sqrt{x} - x)dx$. Hence, the total volume is

$$\int_0^1 2\pi x(\sqrt{x} - x)dx.$$

Of course, it would not be hard to continue and evaluate this integral.

Problem 3. (2 points) Set up an integral (but do not evaluate it) for the length of the curve $y = x^3$ for $1 \leq x \leq 2$.

Solution.

$$\int_1^2 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \int_1^2 \sqrt{1 + (3x^2)^2} dx = \int_1^2 \sqrt{1 + 9x^4} dx$$

Problem 4. (5 points) Solve the initial value problem $\frac{dy}{dx} = y^2$, $y(3) = 1$.

Solution. We separate variables,

$$\frac{1}{y^2} dy = dx$$

and integrate

$$\int \frac{1}{y^2} dy = \int dx$$

to find

$$-\frac{1}{y} = x + C.$$

Plugging in $y = 1$ and $x = 3$, we find $C = -4$. Solving for y , we find that

$$y(x) = -\frac{1}{x - 4} = \frac{1}{4 - x}.$$

Problem 5. (5 points) Evaluate the following indefinite integral: $\int \sin(2t)\cos^5(2t)dt$

Solution. We substitute $u = \cos(2t)$. Since

$$\frac{du}{dt} = -2\sin(2t),$$

we use $\sin(2t) dt = -\frac{1}{2}du$ to get

$$\int \sin(2t)\cos^5(2t)dt = -\frac{1}{2} \int u^5 du = -\frac{1}{12}u^6 + C = -\frac{1}{12}\cos^6(2t) + C.$$

Problem 6. (5 points) Evaluate the integral $\int_0^2 \frac{x^2}{\sqrt{x^3+1}} dx$.

Solution. We substitute $u = x^3 + 1$, so that $du = 3x^2 dx$, and get

$$\int_0^2 \frac{x^2}{\sqrt{x^3+1}} dx = \frac{1}{3} \int_1^9 \frac{1}{\sqrt{u}} du = \left[\frac{2}{3} u^{1/2} \right]_1^9 = 2 - \frac{2}{3} = \frac{4}{3}.$$

Problem 7. (5 points) A conical container of radius 10 ft and height 20 ft is completely filled with water (the tip of the cone is at the bottom). Water weighs 62.4 lb/ft³. Write down an integral for how much work it will take to pump the water to a level of 5 ft above the cone's rim. Do not evaluate the integral.

Solution. Make a sketch! Let us denote with y (in ft) the vertical position in such a way that the tip is at $y=0$ and the rim is at $y=20$. We need to pump the water to the level $y=20+5=25$.

We consider a horizontal slice at height y and thickness dy :

- This slice needs to be lifted up $25 - y$ (ft).
- The slice is (almost) a disk with radius $r = \frac{10}{20}y = \frac{y}{2}$. Hence, its volume is $\pi r^2 dy = \frac{\pi}{4} y^2 dy$ (ft³).
- The weight of the slice is $62.4 \frac{\pi}{4} y^2 dy$ (lb) and it needs to be lifted up $25 - y$ (ft). That takes work of $(25 - y) \cdot 62.4 \frac{\pi}{4} y^2 dy$ (ft-lb).
- “Adding” up, the total work required is

$$\int_0^{20} (25 - y) \cdot 62.4 \frac{\pi}{4} y^2 dy,$$

which is about $1.31 \cdot 10^6$ ft-lb (but we were asked to not evaluate the integral).

(extra scratch paper)